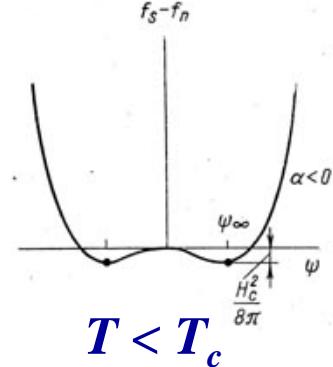
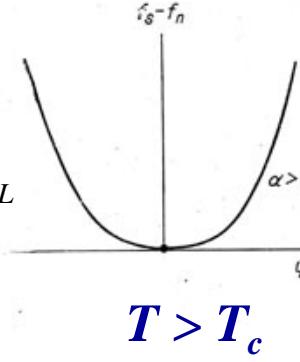


Термические флюктуации в сверхпроводниках.

$$f_s - f_n = f_{GL} = a(T)/Y^2 + 0.5b/Y^4 + /(-i\hbar\tilde{N} - qA)Y^2/2m + m_0H^2/2$$

$$\langle |\Psi|^2 \rangle = \frac{\sum |\Psi|^2 \exp(-\frac{F_{GL}}{k_B T})}{\sum \exp(-\frac{F_{GL}}{k_B T})}$$

$$F_{GL} = \int_V dV f_{GL}$$



Первое уравнение Гинзбурга–Ландау

$$df_{GL}/dY \Rightarrow a(T)Y + b/Y^3 Y + (-i\hbar\tilde{N} - qA)^2 Y/2m = 0$$

Второе уравнение Гинзбурга–Ландау

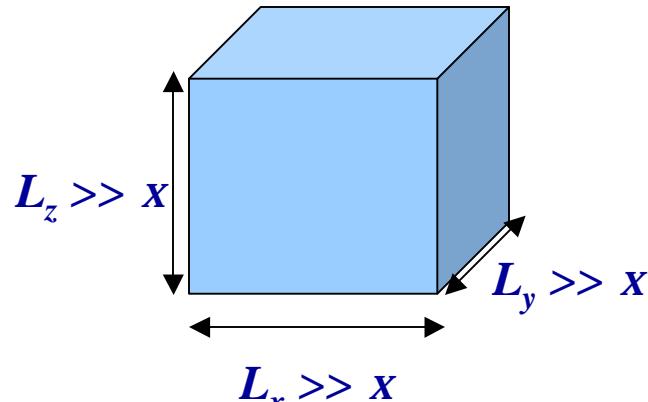
$$df_{GL}/dA \Rightarrow j = (-iq\hbar/2m)(Y^* \tilde{N} Y - Y \tilde{N} Y^*) - (q^2 m_0/m)/Y^2 A$$

Сверхпроводник в отсутствии магнитного поля

$$\frac{F_{GL}}{k_B T} = \frac{\int_V dV f_{GL}}{k_B T} = \frac{\int_V dV (a |\Psi|^2 + 0.5b |\Psi|^4 + \frac{\hbar^2}{2m} |\nabla \Psi|^2)}{k_B T}$$

$$f_{GL} = a(T)/Y^2 + 0.5b/Y^4 \Rightarrow |Y|^2 = 0 \text{ при } T > T_c; |Y|^2 = -a(T)/b \text{ при } T < T_c$$

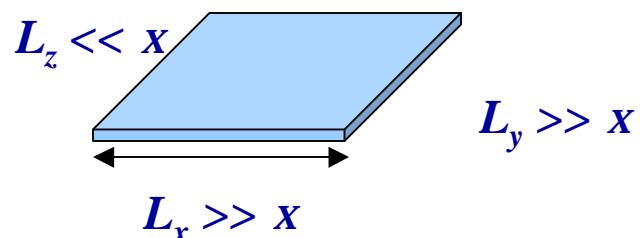
Сверхпроводники с различной размерностью



Трехмерный сверхпроводник: все три размера L_x, L_y, L_z ,
больше корреляционной длины $x = (\hbar^2/2m\alpha)^{1/2}$.

$$\frac{F_{GL}}{k_B T} = \frac{\int_V dx dy dz (a |\Psi|^2 + 0.5b |\Psi|^4 + \frac{\hbar^2}{2m} |\nabla \Psi|^2)}{k_B T}$$

Двумерный сверхпроводник: $L_x, L_y >> x, d = L_z \ll x$.



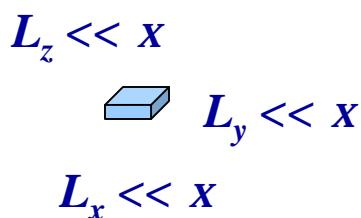
$$\frac{F_{GL}}{k_B T} = \frac{L_z \int_V dx dy (a |\Psi|^2 + b |\Psi|^4 + \frac{\hbar^2}{2m} |\nabla \Psi|^2)}{k_B T}$$

Одномерный сверхпроводник: $L_x >> x, L_y, L_z \ll x. L_y L_z = S$

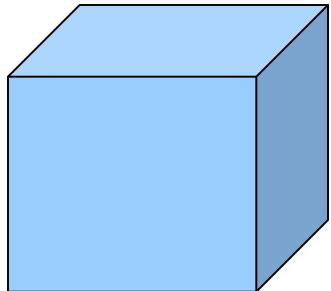
$$\frac{F_{GL}}{k_B T} = \frac{L_y L_z \int_V dx (a |\Psi|^2 + b |\Psi|^4 + \frac{\hbar^2}{2m} |\nabla \Psi|^2)}{k_B T}$$

Ноль-мерный сверхпроводник: $L_x, L_y, L_z \ll x. L_x L_y L_z = V$

$$\frac{F_{GL}}{k_B T} = \frac{L_x L_y L_z (a |\Psi|^2 + b |\Psi|^4 + \frac{\hbar^2}{2m} |\nabla \Psi|^2)}{k_B T}$$



$$\Psi(x, y, z) = \frac{1}{\sqrt{V}} \sum_k \Psi'_k \exp i(\frac{2pk_x}{L_x}x + \frac{2pk_y}{L_y}y + \frac{2pk_z}{L_z}z) \quad k = (k_x, k_y, k_z) \quad q_x = \frac{2pk_x}{L_x} \quad q_y = \frac{2pk_y}{L_y} \quad q_z = \frac{2pk_z}{L_z}$$



$$\int_V dx dy dz |\Psi|^2 = \sum_k |\Psi'_k|^2 \quad \int_V dx dy dz |\nabla \Psi|^2 = \sum_k (q_x^2 + q_y^2 + q_z^2) |\Psi'_k|^2$$

$$\int_V dx dy dz |\Psi|^4 = \frac{1}{V} \sum_{k_i} d(k_1 + k_2 - k_3 - k_4) \Psi'^*_k \Psi'^*_k \Psi'_k \Psi'_k = \frac{1}{V} \sum_{k_i} V_{k_1 k_2 k_3 k_4} \Psi'^*_k \Psi'^*_k \Psi'_k \Psi'_k$$

$$\frac{F_{GL}}{k_B T} = \frac{1}{k_B T} \sum_k [a + \frac{\mathbf{h}^2 q_x^2}{2m} + \frac{\mathbf{h}^2 q_y^2}{2m} + \frac{\mathbf{h}^2 q_z^2}{2m}] |\Psi'_k|^2 + \frac{b}{2V k_B T} \sum_{k_i} V_{k_1 k_2 k_3 k_4} \Psi'^*_k \Psi'^*_k \Psi'_k \Psi'_k$$

$$L_{unit} = \left(\frac{\mathbf{h}^2}{2m} \right)^2 \frac{1}{bk_B T} = x(0) \frac{x^3(0) m_0 H_c^2(0)}{k_B T} = \frac{x(0)}{Gi_{3D}^{1/2}} \quad |\Psi'|^2 = \left(\frac{\mathbf{h}^2}{2m} \right)^3 \frac{1}{b^2 k_B T} = x^3(0) n_s(0) \frac{x^3(0) m_0 H_c^2(0)}{k_B T} |\Psi|^2 = |\Psi|^2 \frac{x^3(0) n_s(0)}{Gi_{3D}^{1/2}}$$

$$\frac{F_{GL}}{k_B T} = \sum_k [\frac{T/T_c - 1}{Gi_{3D}} + q^2] |\Psi_k|^2 + \frac{1}{2V} \sum_{k_i} V_{k_1 k_2 k_3 k_4} \Psi'^*_k \Psi'^*_k \Psi'_k \Psi'_k \quad q^2 = q_x^2 + q_y^2 + q_z^2 \quad Gi_{3D} = \left(\frac{k_B T}{x^3(0) m_0 H_c^2(0)} \right)^2$$

Число Гинзбурга $\textcolor{blue}{Gi}$ определяет ширину области флюктуаций вблизи $\textcolor{blue}{T}_c$.

$$\textcolor{blue}{Nb} \quad \textcolor{blue}{T}_c = 9.2 \text{ K}, m_0 H_c(0) = 0.2 \text{ T}, x(0) = 40 \text{ nm} \quad \textcolor{blue}{P} \quad Gi_{3D} = 2.5 \cdot 10^{-9}$$

$$\textcolor{blue}{YBa}_2\textcolor{blue}{Cu}_3\textcolor{blue}{O}_{7-x} \quad \textcolor{blue}{T}_c = 93 \text{ K}, m_0 H_c(0) = 2 \text{ T}, x(0) = 1.2 \text{ nm} \quad \textcolor{blue}{P} \quad Gi_{3D} = 0.1$$

$$m_0 H_c^2(0) = 3.02 N(0) (k_B T_c)^2$$

$$Gi_{3D} = \left(\frac{k_B T_c}{x^3(0) m_0 H_c^2(0)} \right)^2 = \left(\frac{1}{x^3(0) 3.02 N(0) k_B T_c} \right)^2$$

Двумерный сверхпроводник: $\mathbf{L}_x, \mathbf{L}_y \gg x, d = L_z \ll x$.

$$\frac{F_{GL}}{k_B T} = \sum_k \left[\frac{T/T_c - 1}{Gi_{2D}} + q^2 \right] |\Psi_k|^2 + \frac{1}{2S} \sum_{k_i} V_{k_1 k_2 k_3 k_4} \Psi_{k_1}^* \Psi_{k_2}^* \Psi_{k_3} \Psi_{k_4}$$



$$q^2 = q_x^2 + q_y^2 \quad Gi_{2D} = \frac{k_B T}{d x^2(0) m_0 H_c^2(0)} = \frac{x(0)}{d} Gi_{3D}^{1/2}$$

$$L_{unit} = \frac{x(0)}{Gi_{2D}^{1/2}} \quad |\Psi'|^2 = |\Psi|^2 x^2(0) n_s(0)$$

Одномерный сверхпроводник: $\mathbf{L}_x \gg x, \mathbf{L}_y, \mathbf{L}_z \ll x. \mathbf{L}_y \mathbf{L}_z = S$

$$\frac{F_{GL}}{k_B T} = \sum_k \left[\frac{T/T_c - 1}{Gi_{1D}} + q^2 \right] |\Psi_k|^2 + \frac{1}{2L} \sum_{k_i} V_{k_1 k_2 k_3 k_4} \Psi_{k_1}^* \Psi_{k_2}^* \Psi_{k_3} \Psi_{k_4}$$



$$q^2 = q_x^2 \quad Gi_{1D} = \left(\frac{k_B T}{s x(0) m_0 H_c^2(0)} \right)^{2/3} = \left(\frac{x^2(0)}{s} \right)^{2/3} Gi_{3D}^{1/3}$$

$$L_{unit} = \frac{x(0)}{Gi_{1D}^{1/2}} \quad |\Psi'|^2 = |\Psi|^2 x^2(0) n_s(0) Gi_{1D}^{1/2}$$

Ноль-мерный сверхпроводник: $\mathbf{L}_x, \mathbf{L}_y, \mathbf{L}_z \ll x. \mathbf{L}_x \mathbf{L}_y \mathbf{L}_z = V$



$$\frac{F_{GL}}{k_B T} = \frac{T/T_c - 1}{Gi_{0D}} |\Psi|^2 + \frac{1}{2} |\Psi|^4 \quad Gi_{0D} = \left(\frac{k_B T}{V m_0 H_c^2(0)} \right)^{2/3} = \left(\frac{x^3(0)}{V} \right)^{1/4} Gi_{3D}^{1/4}$$

$$Nb \quad Gi_{3D} = 2.5 \cdot 10^{-9} P \quad Gi_{0D} = Gi_{3D}^{1/4} = 0.7 \cdot 10^{-2}$$

$$|\Psi'|^2 = |\Psi|^2 n_s(0) Gi_{0D} = \Psi^2 n_s(0) Gi_{0D} \quad \frac{T/T_c - 1}{Gi_{0D}} = e_{0D} \quad \frac{F_{GL}}{k_B T} = e_{0D} \Psi^2 + 0.5 \Psi^4$$



$$\langle \Psi^2 \rangle = \frac{\sum \Psi^2 \exp(-\frac{F_{GL}}{k_B T})}{\sum \exp(-\frac{F_{GL}}{k_B T})} = \frac{\int_0^\infty d\Psi^2 \Psi^2 \exp(-e_{0D} \Psi^2 - 0.5 \Psi^4)}{\int_0^\infty d\Psi^2 \exp(-e_{0D} \Psi^2 - 0.5 \Psi^4)}$$

Б.В.Шмидт, 1967

Линейное приближение. $T > T_c$; $e_{0D} \gg 1$; $T/T_c - 1 \gg Gi_{0D}$

$$\langle \Psi^2 \rangle = \frac{\int_0^\infty d\Psi^2 \Psi^2 \exp(-e_{0D} \Psi^2 - 0.5 \Psi^4)}{\int_0^\infty d\Psi^2 \exp(-e_{0D} \Psi^2 - 0.5 \Psi^4)} \approx \frac{\int_0^\infty d\Psi^2 \Psi^2 \exp(-e_{0D} \Psi^2)}{\int_0^\infty d\Psi^2 \exp(-e_{0D} \Psi^2)} = \frac{1}{e_{0D}}$$

Приближение Хартри-Фока. $T > T_c$; $e_{0D} \gg 1$; $T/T_c - 1 \gg Gi_{0D}$ $\Psi^4 \approx \langle \Psi^2 \rangle \Psi^2 + \Psi^2 \langle \Psi^2 \rangle$

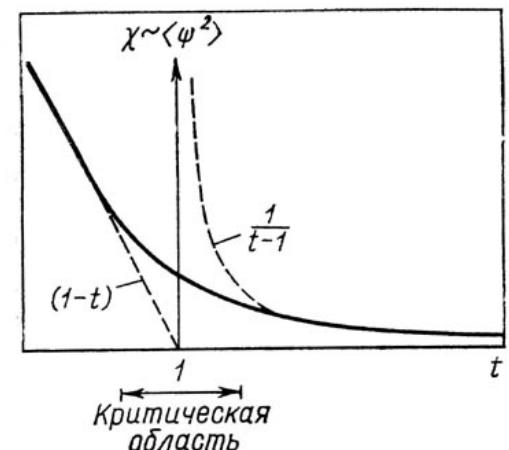
$$\langle \Psi^2 \rangle \approx \frac{\int_0^\infty d\Psi^2 \Psi^2 \exp(-e_{0D} \Psi^2 + \langle \Psi^2 \rangle \Psi^2)}{\int_0^\infty d\Psi^2 \exp(-e_{0D} \Psi^2 + \langle \Psi^2 \rangle \Psi^2)} = \frac{1}{e_{0D} + \langle \Psi^2 \rangle}$$

$$\langle \Psi^2 \rangle^2 + e_{0D} \langle \Psi^2 \rangle - 1 = 0 \rightarrow \langle \Psi^2 \rangle = -\frac{e_{0D}}{2} + \sqrt{\left(\frac{e_{0D}}{2}\right)^2 + 1}$$

при $T > T_c$; $e_{0D} \gg 1 \Rightarrow \langle Y^2 \rangle \gg 1/e_{0D} = Gi_{0D}/(T/T_c - 1)$

при $T < T_c$; $-e_{0D} \gg 1 \Rightarrow \langle Y^2 \rangle \gg -e_{0D} = (1 - T/T_c)/Gi_{0D}$

$$\langle |\Psi'|^2 \rangle = \langle \Psi^2 \rangle n_s(0) Gi_{0D} = -e_{0D} n_s(0) Gi_{0D} = n_s(0) \left(1 - \frac{T}{T_c}\right)$$



При фазовом переходе возникает дальний порядок и функции описывающие температурные зависимости термодинамических величин имеют особенность.

$$F = -k_B T \ln \left(\sum |\Psi|^2 \exp(-\frac{F_{GL}}{k_B T}) \right) \quad S = -\frac{\partial F}{\partial T} = -\frac{\partial a}{\partial T} \frac{\sum |\Psi|^2 \exp(-\frac{F_{GL}}{k_B T})}{\sum \exp(-\frac{F_{GL}}{k_B T})} = -\frac{a_0}{T_c} \langle |\Psi|^2 \rangle$$

$$C = T \frac{\partial S}{\partial T} = -T \frac{a_0}{T_c} \frac{\partial \langle |\Psi|^2 \rangle}{\partial T}$$

λ - особенность и скачок теплоемкости

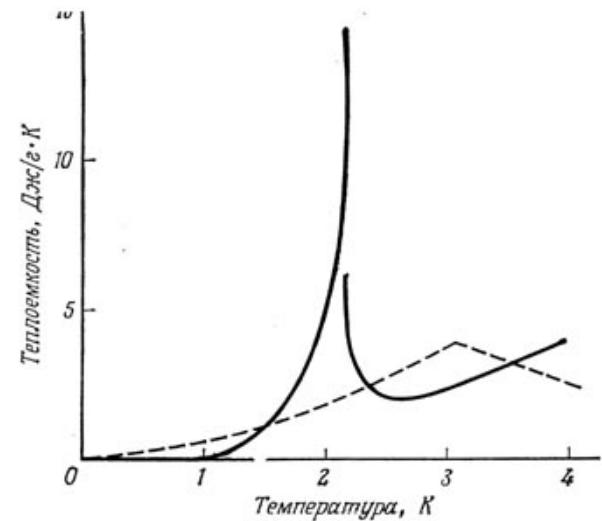
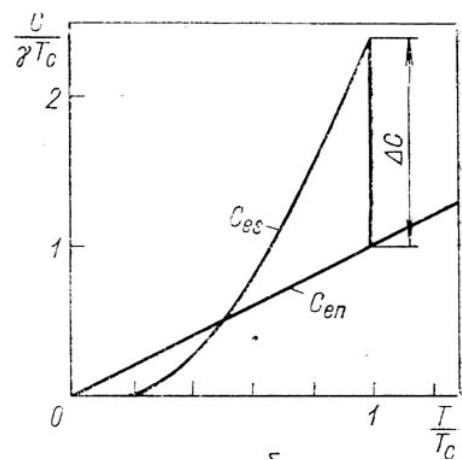
$/Y^2 = 0$ при $T > T_c$; $/Y^2 = -a(T)/b$ при $T < T_c$

$C = 0$ при $T > T_c$; $C = -Ta_0^2/T_c^2b$ при $T < T_c$

Скачек теплоемкости не зависит от размерности

$\Delta C = -a_0^2/T_c b$ при **3D, 2D, 1D** и **0D**

Nb 3D: $Gi_{3D} = 2.5 \cdot 10^{-9}$; Nb 0D: $Gi_{0D} = 0.7 \cdot 10^{-2}$

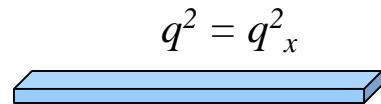


Линейное приближение. $T > T_c$; $e \gg 1$; $T/T_c - 1 \gg Gi$

$$\frac{F_{GL}}{k_B T} = \sum_k \left[\frac{T/T_c - 1}{Gi_{nD}} + q^2 \right] |\Psi_k|^2 + \frac{1}{2V} \sum_{k_i} V_{k_1 k_2 k_3 k_4} \Psi_{k_1}^* \Psi_{k_2}^* \Psi_{k_3} \Psi_{k_4} \approx \sum_k (e_{nD} + q^2) |\Psi_k|^2$$

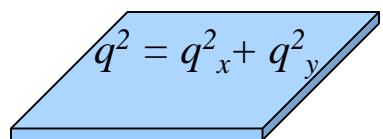
$$\langle |\Psi_k|^2 \rangle = \frac{\sum_k |\Psi_k|^2 \exp(-\frac{F_{GL}}{k_B T})}{\sum_k \exp(-\frac{F_{GL}}{k_B T})} = \frac{\int_0^\infty d|\Psi_k|^2 |\Psi_k|^2 \exp(-(e_{nD} + q^2)) |\Psi_k|^2}{\int_0^\infty d|\Psi_k|^2 \exp(-(e_{nD} + q^2)) |\Psi_k|^2} = \frac{1}{e_{nD} + q^2}$$

$$\int_V dV |\Psi|^2 = \sum_k |\Psi_k|^2 \quad \longrightarrow \quad \langle |\Psi|^2 \rangle = \frac{1}{V} \int_V dV |\Psi|^2 = \frac{1}{V} \sum_k |\Psi_k|^2 = \frac{1}{V} \sum_k \frac{1}{e_{nD} + q^2} = \frac{1}{(2p)^n} \int_q \frac{d^n q}{e_{nD} + q^2}$$



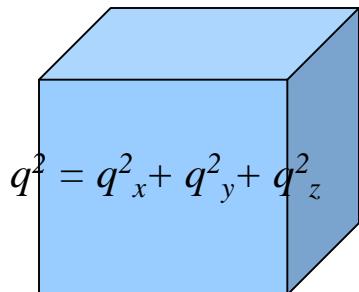
$$\langle |\Psi|^2 \rangle = \frac{1}{L_x} \sum_k \frac{1}{e_{1D} + q_x^2} = \frac{1}{2p} \int_{-\infty}^{\infty} \frac{dq_x}{e_{1D} + q_x^2}$$

$$L_x \gg L_{unit} = \frac{x(0)}{Gi_{1D}^{1/2}}$$



$$\langle |\Psi|^2 \rangle = \frac{1}{L_x L_y} \sum_k \frac{1}{e_{1D} + q_x^2 + q_y^2} = \frac{1}{(2p)^2} \iint \frac{dq_x dq_y}{e_{1D} + q_x^2 + q_y^2}$$

$$q_x = \frac{2pk_x}{L_x} \ll 2pGi_{nD}^{1/2}$$



$$\langle |\Psi|^2 \rangle = \frac{1}{L_x L_y L_z} \sum_k \frac{1}{e_{1D} + q_x^2 + q_y^2 + q_z^2} = \frac{1}{(2p)^3} \iiint \frac{dq_x dq_y dq_z}{e_{1D} + q_x^2 + q_y^2 + q_z^2}$$



Одномерный сверхпроводник: $L_x \gg x, L_y, L_z \ll x, L_y L_z = S$

Линейное приближение. $T > T_c; e_{ID} \gg 1; T/T_c - 1 \gg Gi_{ID}$

$$\frac{F_{GL}}{k_B T} \approx \sum_k (e_{1D} + q^2) |\Psi_k|^2 \quad \rightarrow \quad \langle |\Psi|^2 \rangle = \frac{1}{2p} \int_{-\infty}^{\infty} \frac{dq_x}{e_{1D} + q_x^2} = \frac{1}{2p e_{1D}^{1/2}} \int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \frac{1}{2e_{1D}^{1/2}}$$

Приближение Хартри-Фока. $T > T_c; e_{ID} \gg 1; T/T_c - 1 \approx Gi_{1D}$

$$\int_{L_x} dx |\Psi|^4 = \frac{1}{L_x} \sum_{k_i} d(k_1 + k_2 - k_3 - k_4) \Psi_{k_1}^* \Psi_{k_2}^* \Psi_{k_3} \Psi_{k_4} \approx \langle |\Psi|^2 \rangle \sum_k |\Psi_k|^2 + \langle |\Psi|^2 \rangle \sum_k |\Psi_k|^2 \langle |\Psi|^2 \rangle$$
$$\frac{F_{GL}}{k_B T} \approx \sum_k (e_{1D} + \langle |\Psi|^2 \rangle + q^2) |\Psi_k|^2 \quad \rightarrow \quad \langle |\Psi|^2 \rangle = \frac{1}{2(e_{1D} + \langle |\Psi|^2 \rangle)^{1/2}}$$

при $T > T_c; e_{ID} \gg 1 \Rightarrow \langle |Y|^2 \rangle \gg 1/2 e_{ID}^{1/2} = Gi_{1D}^{1/2} / (T/T_c - 1)^{1/2}$

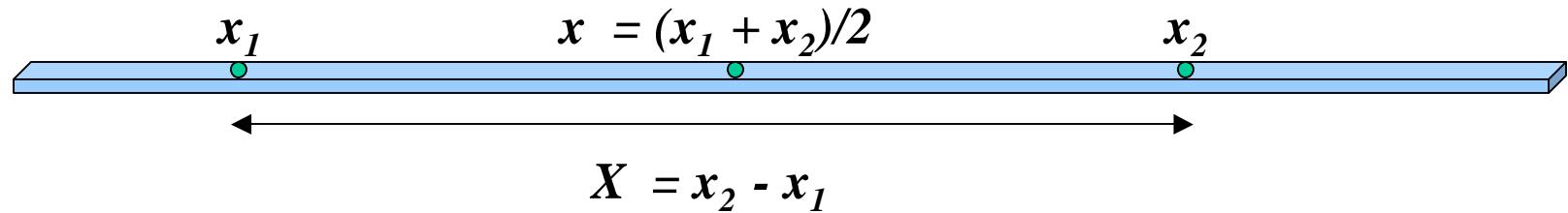
при $T < T_c; -e_{ID} \gg 1 \Rightarrow \langle |Y|^2 \rangle \gg -e_{ID} = (1 - T/T_c) / Gi_{1D}$

Функции, описывающие температурные зависимости термодинамических величин одномерного сверхпроводника, не имеют особенностей.

Дальний порядок и фазовом переходе невозможны в одномерных системах.

Корреляционная функция. $\mathbf{T} > \mathbf{T}_c$

$$g(x_1, x_2) = \langle \Psi^*(x_1) \Psi(x_2) \rangle = \left\langle \frac{1}{\sqrt{L_x}} \sum_{k_1} \Psi_{k_1} * \exp(-iq_1 x_1) \frac{1}{\sqrt{L_x}} \sum_{k_2} \Psi_{k_2} \exp(iq_2 x_2) \right\rangle$$



$$g(x_1, x_2) = \left\langle \frac{1}{L_x} \sum_{k_1 k_2} \Psi_{k_1} * \Psi_{k_2} \exp[i0.5(q_1 + q_2)X] \exp[-i(q_1 - q_2)x] \right\rangle = \sum_k \langle |\Psi_k|^2 \rangle \exp[iqX] = g(X)$$

Линейное приближение. $\mathbf{T} > \mathbf{T}_c$

$$g(X) = \frac{1}{L_x} \sum_k \langle |\Psi_k|^2 \rangle \exp(iqX) = \frac{1}{2p} \int_{-\infty}^{\infty} \frac{dq \exp(iqX)}{e_{1D} + q^2} \propto \exp(-e_{1D}^{1/2} X) \quad \frac{T/T_c - 1}{Gi_{1D}} = e_{1D} \quad L_{unit} = \frac{x(0)}{Gi_{1D}^{1/2}}$$

Корреляционная длина $l_{cor} = 1/e_{1D}^{1/2} = Gi_{1D}^{1/2}/(T/T_c - 1)^{1/2}$ при $\mathbf{T} > \mathbf{T}_c$
 $l_{cor} = x(0)/(T/T_c - 1)^{1/2} = x(T); x(T) = h/(2m|a|)^{1/2} = h/(2ma_0)^{1/2} / |T/T_c - 1|^{1/2}$

Приближение Хартри-Фока. $\mathbf{T} > \mathbf{T}_c$

$$g(X) = \frac{1}{2p} \int_{-\infty}^{\infty} \frac{dq \exp(iqX)}{e_{1D} + \langle |\Psi|^2 \rangle + q^2} \propto \exp[-(e_{1D} + \langle |\Psi|^2 \rangle)^{1/2} X]$$

$l_{cor} = 1/(e_{1D} + \langle |Y|^2 \rangle)^{1/2} = 2\langle |Y|^2 \rangle^{-1/2}$ при всех \mathbf{T}

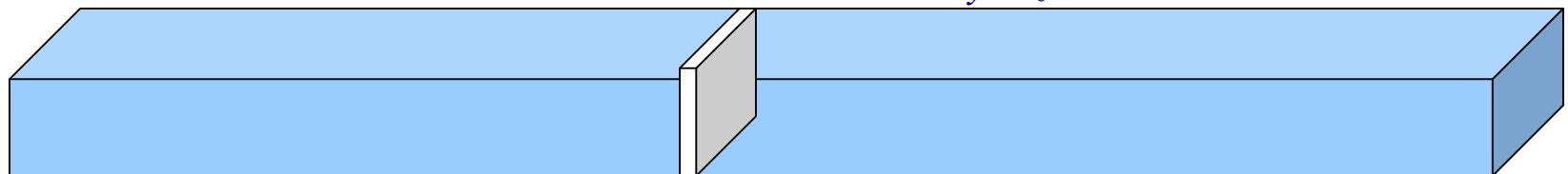
Центры проскальзывания фазы. $T < T_c$

$$x \quad L_y, L_z < x$$



$$P \propto \exp(-e_{0D}\Psi^2 + 0.5\Psi^4) \quad P_{\max} \propto \exp(0.5e_{0D}^2) \quad P(\Psi = 0) \propto \exp(0) = 1$$

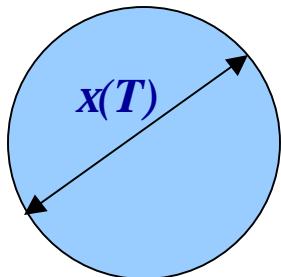
$$x \quad L_y, L_z > x$$



$$\frac{P_{\max}}{P(\Psi = 0)} = \exp\left(\frac{s}{x^2} 0.5 e_{0D}^2\right)$$

Трехмерный сверхпроводник. $T > T_c$

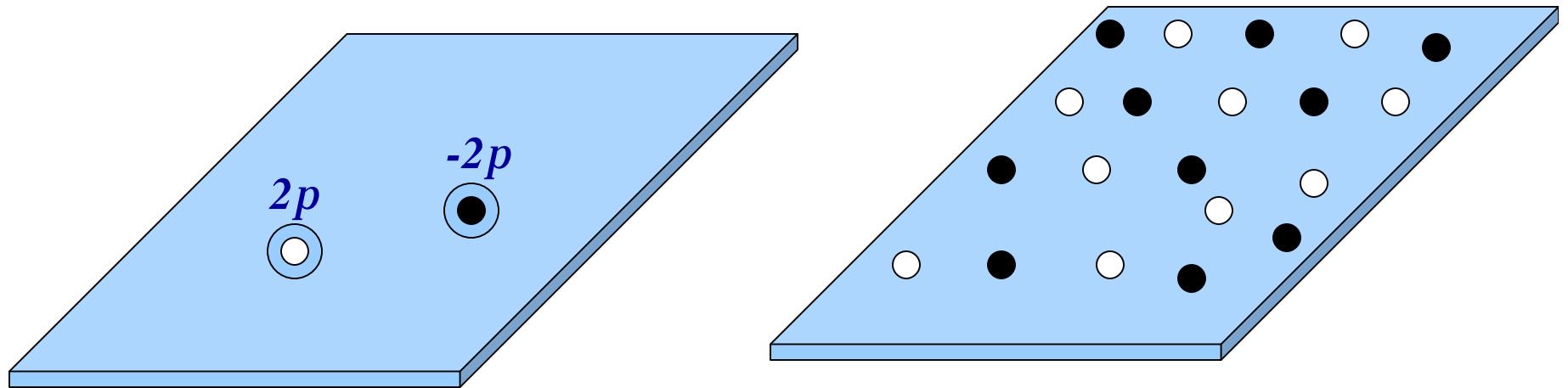
$$g(X) = \left(\frac{mk_B T}{2p\hbar^2} \right) \frac{\exp[-\frac{R}{X(T)}]}{R}$$



$$x(T) = x(0)/(T/T_c - 1)^{1/2} \text{ при } e_{3D} \gg 1; T/T_c - 1 \gg Gi_{3D}$$

$$x(T) = x(0)/(T/T_c - 1)^a \text{ при } e_{3D} \ll 1; T/T_c - 1 \ll Gi_{3D}$$

Двумерный сверхпроводник. $T < T_c$ Переход Костерлица-Таулиса.



$$\oint_l dl \nabla j = 2p\mathbf{h} \quad \bigcirc \quad \oint_l dl \nabla j = -2p\mathbf{h} \quad \bullet$$

$$\oint_l dl I_L^2 j_s = \frac{\Phi_0}{2p} \oint_l dl \nabla j - \Phi = \Phi_0 - \Phi$$